Data analysis using a combination of independent component analysis and empirical mode decomposition

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A combination of independent component analysis and empirical mode decomposition (ICA-EMD) is proposed in this paper to analyze low signal-to-noise ratio data. The advantages of ICA-EMD combination are these: ICA needs few sensory clues to separate the original source from unwanted noise and EMD can effectively separate the data into its constituting parts. The case studies reported here involve original sources contaminated by white Gaussian noise. The simulation results show that the ICA-EMD combination is an effective data analysis tool.

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I. INTRODUCTION

Data analysis is indispensable for scientific research, but the available "data processing" methods cannot satisfy the need to analyze data from nonstationary and nonlinear processes. Yet, data analysis is crucial in research as it paves the way for discovery in physics and nature. We urgently need new tools for new data and new discoveries. The development of the empirical mode decomposition (EMD) and ensemble EMD methods [1,2] had alleviated the need for new tools to a large. EMD has been applied in many physicals fields and contributed to the advancement of the science [3–10]. But the EMD method, by itself, could not separate signal from high-intensity noise, for the method treat the existing noise as part of the data. Independent component analysis (ICA) on the other hand could separate the unwanted noise from the data, if enough sensors are available. Therefore, we propose here to couple the two methods using ICA as a preprocessor to remove the noise and then employ EMD or EEMD to decompose the cleaned data into their constituting components.

ICA is a recently developed decomposition synthesizing technique that combines mathematics, statistics, physics, and computer simulations. Jutten and Herault [11] developed ICA in the early 1990s to solve the cocktail-party problem. Bell and Sejnowski [12] used it to develop their model based on the principle of information maximization preservation. A couple of years later the fixed-point or FastICA algorithm was devised [13]. Briefly, ICA is a method for finding underlying factors or components in multivariate (multidimensional) statistical data, based on their statistical independence [14,15]. ICA has been applied in many fields, including medicine [16], fMRI (functional magnetic resonance imaging) [17], optics [18,19], image recognition [20], machine learning [21], sound analysis [22], phase synchronization [23], complex systems [24], Fermilab Boosters [25], and in EMG (magnetoencephalography) and ECG (electrocardiogram)[26,27] .

EMD is a data analysis technique similar to wavelet analysis and singular spectrum analysis (SST); it is particularly suitable for nonlinear and nonstationary time series analysis. However, EMD does not assume an *a priori* basis. Unlike wavelet analysis and SST, EMD is suitable for describing nonlinear phenomena. In 1998, Huang [1] proposed an EMD method for analyzing nonlinear and nonstationary data. This decomposition method is adaptive and highly efficient. Huang [2,28,29] also developed intermittence and ensemble EMD. In 1998 Huang *et al.* [30] applied EMD to the study of blood pressure waves in the lung. In 2004 Cummings *et al.* [31] used EMD to look at the traveling waves associated with the occurrence of dengue hemorrhagic fever in Thailand. In 2004 Balocchi *et al.* [7] applied EMD to analyze heartbeat intervals series. In 2005 Lam *et al.* [8] applied EMD to the measurement of hurst exponents for semiconductor laser phase dynamics. Moreover, in 2007, Kozakov *et al.* [9] proposed EMD for the analysis of the obtained spatiotemporal patterns. Camp *et al.* [10] employed EMD and the Hilbert transform to search for gravitational waves. In 2007 Wu and Huang [2,32] proposed EMD/EEMD methods as robust decomposition tools that also serves as a solution to mode mixing problems. Although EEMD is designed on the premise that noise could assist us in data analysis, EEMD by itself still could not separate the original noise from the data.

Although ICA has many advantages in data analysis, it has a big disadvantage: The number of sensors should be greater than or equal to the number of sources [33–35]. At present it has been found that with the development of overcomplete or undercomplete representations [33–37], all of the original data cannot be separated with a small number of sensors. Despite EMD's wide application for data analysis in many fields, it still has the problem of analysis data under low-signal-to-noise-ratio (SNR) conditions. To compensate for the weaknesses of both ICA and EMD and to achieve optimal effectiveness, we introduce the combined ICA-EMD method, which combines the advantages of EMD with those of ICA. McKeown *et al.* also proposed the combination of ICA and EMD for processing biomedical signals in EMG and EEG [38,39]. However, they did not apply the method to low-SNR environment with small number of sensors. Many

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applications in natural sciences and engineering take place in a low-SNR environment and the number of sensors allowed for each application may vary. Moreover, in these two studies, neither was ICA employed for the removal of noise nor EMD for source separation. They both used EMD as a filter to identify the designated signals they needed between 10 and 30 Hz. The ICA-EMD method proposed here can be used to separate original data and ambient noise with only two sensors in low-SNR conditions. Before presenting the applications of this combined method, brief summaries of ICA and EMD will be given first.

II. SUMMARY ON ICA

ICA is a method for solving the problem of noise source separation. Hence we will use ICA to reject the white Gaussian noise. Assume that there is an M-dimensional zero-mean unknown underling or original sources $\mathbf{s}(t)=[s_1(t),\cdots,s_M(t)]^T$, that the components of s_i are mutually independent, and that one of $s_1(t),\cdots,s_M(t)$ is a white Gaussian noise. The unknown underling sources $\mathbf{s}(t)$ is equivalent to M-independent scalar-valued source data $s_i(t)$. All the source data must be statistically independent, and have a non-Gaussian probability distribution. An exception is allowed for only one component. We can write the multivariate probability density function of the vector as the product of marginal independent distributions

$$p(\mathbf{s})=\prod_{i=1}^{M}p_i(s_i). \tag{1}$$

The measured data $\mathbf{x}(t)=[x_1(t),\ldots,x_N(t)]^T$ are observed at each time point t so that

$$\mathbf{x}(t)=\mathbf{A}\mathbf{s}(t). \tag{2}$$

The source data, $\mathbf{s}(t)$, becomes the observed ones through a transform represented by a M-by-N matrix $\mathbf{A}$,

$$\mathbf{A}=\begin{bmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NM} \end{bmatrix}, \tag{3}$$

where the $\mathbf{A}$ matrix consists of some parameters that depend on the scale of the sensors from the underling sources. However, with ICA, we can retrieve the statistically independent signals by estimating an unmixing matrix $\mathbf{W}$. The aim of ICA is to estimate the matrix in such a way that $\mathbf{W}\approx\mathbf{A}^{-1}$. Then, the $\mathbf{W}$ matrix can be used to retrieve the statistically independent signals. The equation for signal retrieval is

$$\mathbf{u}(t)=\mathbf{W}\mathbf{x}(t)=\mathbf{W}\mathbf{A}\mathbf{s}(t)\approx\mathbf{s}(t), \tag{4}$$

where $\mathbf{u}(t)=[u_1(t),\ldots,u_N(t)]^T$ are estimate of the underling sources. ICA is intended to estimate the actual $\mathbf{W}=\mathbf{A}^{-1}$.

In fact, we would not be able to know how many underling sources there are in a real-life situation. Consequently, the number of sensors measuring the underling sources is also unknown. ICA's statistical independence can be measured by the entropy. Entropy is a basic concept of information theory [40,41]. Since noise entropy is different from the entropy of mixed original sources, it can be shown that, with only two sensors, noise can be separated from mixed original sources by ICA.

III. SUMMARY ON EMD

Once the underlying source is determined, EMD can be used to reduce the original sources into their constituent components, known as the intrinsic mode functions (IMFs). EMD uses an adaptive basis derived from each data set to decompose the variance of that set into a finite number of IMFs from which instantaneous frequency could be computed through the Hilbert transform. An IMF is a function that satisfies two conditions [1]: (1) the number of extremes and the number of zero crossings in the whole data set must either be equal to or differ at most by one and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. To obtain an IMF from the original signal, Huang [1] suggested the sifting process described below. The sifting process begins with the identification of the local minima and maxima of a time series, $X(t)$. First, identify all the local maxima, and then connect them with a cubic spline line to form the upper envelope $e_u(t)$. Repeat the procedure for the local minima to produce the lower envelope $e_l(t)$.

The local mean can be calculated as shown

$$m_1(t)=\frac{e_u(t)+e_l(t)}{2}. \tag{5}$$

The mean is designated in Eq. (5) and the difference between the data and $m_1(t)$ is the first component $h_1(t)$, as obtained in the following equation:

$$h_1(t)=x(t)-m_1(t). \tag{6}$$

In the subsequent sifting process, $h_1(t)$ is considered the data

$$h_{1(k-1)}(t)-m_{1k}(t)=h_{1k}. \tag{7}$$

EMD can repeat this sifting procedure k times, until h_{1k} is an IMF. Now

$$c_1=h_{1k}, \tag{8}$$

which is the first IMF component derived from the data. The standard deviation determines a criterion for stopping the sifting process. This can be accomplished by limiting the size of the standard deviation (SD), defined and computed from the two consecutive sifting results as follows:

$$\mathrm{SD}=\sum_{t=0}^{T}\left\{\frac{|[h_{1(k-1)}(t)-h_{1k}(t)]|^2}{h_{1(k-1)}^2(t)}\right\}. \tag{9}$$

When the SD can be set between 0.2 and 0.3, the first IMF c_1 is obtained, which can be written as

$$X(t)-c_1=r_1. \tag{10}$$

Note that the residue r_1 still contains some useful information. We can therefore treat the residue as new data and apply the above procedure to obtain

$$r_1-c_1=r_2$$

$$\vdots$$

$$r_{n-1} - c_n = r_n. \tag{11}$$

This procedure should be repeated until the final series r_n no longer carries any oscillation data. The remaining series is the trend of this nonstationary data $X(t)$. Combining Eqs. (10) and (11) yields the EMD of the original signal,

$$X(t) = \sum_{j=1}^{n} c_j + r_n. \tag{12}$$

Thus, one can achieve a decomposition of the data into n-empirical modes and a residue r_n, which can be either the mean trend or a constant. The IMFs $c_1, c_2, \ldots, c_n$ include different frequency bands ranging from high to low.

IV. SIMULATIONS AND RESULTS

A. Application of combined ICA-EMD to low-SNR simulated data

The cocktail-party problem is the most famous example of an ICA application [13,33–35]. There are five different positions s_1, s_2, s_3, s_4, and s_5 with four original sources which are contaminated by white Gaussian noise. In the simulation, $s_1(t)$ represents the white Gaussian noise. $s_2(t)$, $s_3(t)$, $s_4(t)$, and $s_5(t)$ are the 150, 100, and 40 Hz sine waveforms and 3 Hz triangular waveform, respectively. When the five sources occur simultaneously, two microphones are installed at different positions to record sounds. Only two sensors are employed to measure the five sources. The two microphones record signals which constitute a weighted sum of the speech signals emitted by the five speakers, which we denote by s_1, s_2, s_3, s_4, and s_5. The signals received by the two microphones may be represented as follows:

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t) + a_{13}s_3(t) + a_{14}s_4(t) + a_{15}s_5(t),$$

$$x_2(t) = a_{21}s_1(t) + a_{22}s_2(t) + a_{23}s_3(t) + a_{24}s_4(t) + a_{25}s_5(t), \tag{13}$$

where the $a_{ij}(i=1,2,\ j=1,\ldots,5)$ are different parameters that depend on the distance of the microphones from the speakers, and a_{ij} indicates the elements of the $\mathbf{A}$ matrix. These are randomly selected constants. Let us take the following mixing matrix:

$$\mathbf{A} = \begin{bmatrix} 0.6 & 0.4 & 0.8 & 1 & 2 \\ 0.8 & 1 & 0.9 & 0.8 & -4 \end{bmatrix}. \tag{14}$$

However, the mixed matrix $\mathbf{A}$ is usually unknown in our study. Figure 1 shows the time domain of the data received by two microphones. Figure 2 shows the resulting EMD components with straightforward application of the numerical simulation. EMD cannot separate the noise from the signal and decompose the signal into their constituting components under low-SNR condition. ICA, however, finds a linear transformation $\mathbf{W}$ of the dependent sensor waveform $\mathbf{x}$. From Eq. (4) we get the matrix

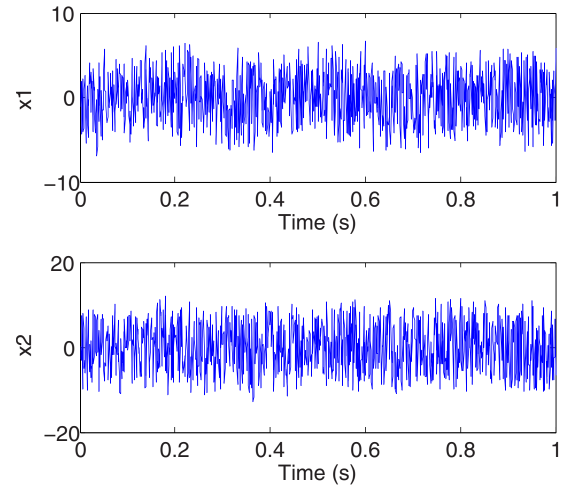

FIG. 1. (Color online) The numerical simulation of data received by two microphones.

$$\mathbf{W} = \begin{bmatrix} 0.1119 & -0.1192 \\ 0.6349 & 0.3142 \end{bmatrix}. \tag{15}$$

The multiplication of matrix $\mathbf{W}$ with $\mathbf{x}$, as in Eq. (4), gives us the ICA output $\mathbf{u}$. The results for the estimation of $\mathbf{u}$ via ICA can be seen in Fig. 3. One is noise and the other comes from mixed sources. With the signal and noise successfully separated, we can use EMD method to decompose data from these mixed sources into its constituting components. Figure 4 shows the resulting ICA-EMD components obtained from the data. The process of EMD reduces the time series under analysis into components, such as IMFs, thereby "sifting" or separating out the different frequency scales of the data. The sifting is done adaptively, without *a priori* structure imposed on the data. The sifting first identifies and removes the components with the highest frequencies, then does the same for lower frequencies down to the lowest trends, as show in Figs. 4(a)–4(e). The original data is presented in Fig. 5. The result indicates that ICA-EMD ensures effective retrieval of original data.

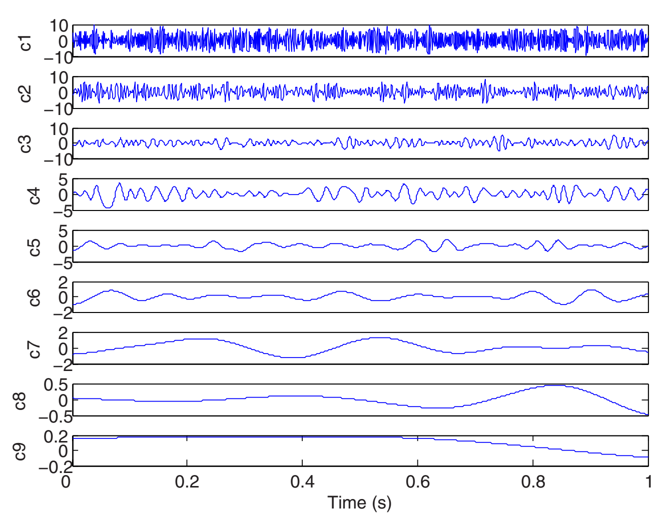

FIG. 2. (Color online) The resulting EMD component from the numerical simulation of data.

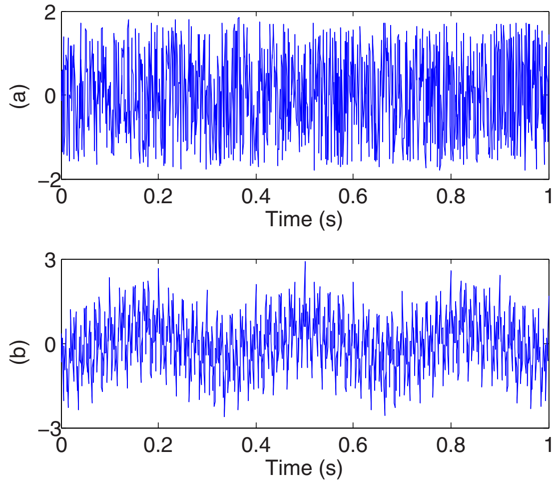

FIG. 3. (Color online) Data separated by ICA: (a) white Gaussian noise; (b) mixed sources.

B. Application of combined ICA-EMD to low-SNR length-of-day data

Let us consider how this method is used to analyze different data using as an example the daily length-of-day (LOD) data set, produced by Gross (2001), which covers the period from 20 January 1962 to 6 January 2001, a total of 14 232 days [42]. The raw data were derived from independent earth-rotational measurements taken by space-geodetic techniques that include lunar and satellite laser ranging, very-long-baseline interferometry, global positioning system (GPS), and optical astronometric measurements. Prior to the recent ensemble EMD [2], Huang [28] developed an intermittent EMD method for extracting the movements and periodic variations caused by the sun, the moon, and the earth. Under some conditions, environmental noise from space may interfere with GPS transmissions and affect the measurements. Solar activities such as sunspots, lonospheric storms, and solar transit are the main sources of disturbance for satellites. Solar transit refers to the situation when the sun, the survey satellite, and the ground station data receiving antenna form a straight line. The electromagnetic waves emitted from the sun interfere with the reception of satellite signals at the ground station, resulting in the interruption of communication. The simulation signals received by the two satellites may be represented as follows:

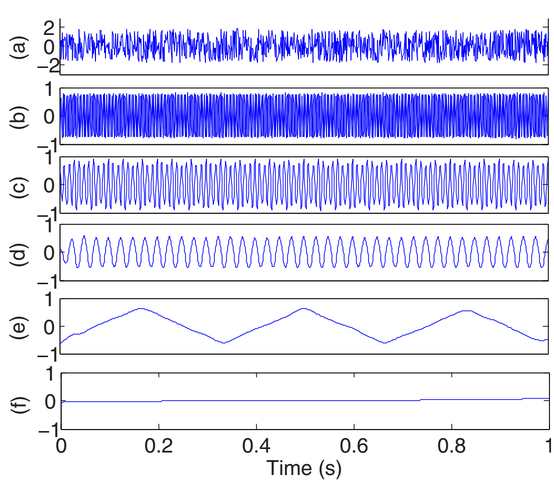

FIG. 4. (Color online) Data separated by ICA-EMD: (a) white Gaussian noise; (b) 150 Hz sin waveform; (c) 100 Hz sin waveform; (d) 40 Hz sin waveform; (e) 3 Hz triangular waveform; (f) the last row corresponding to the final residue.

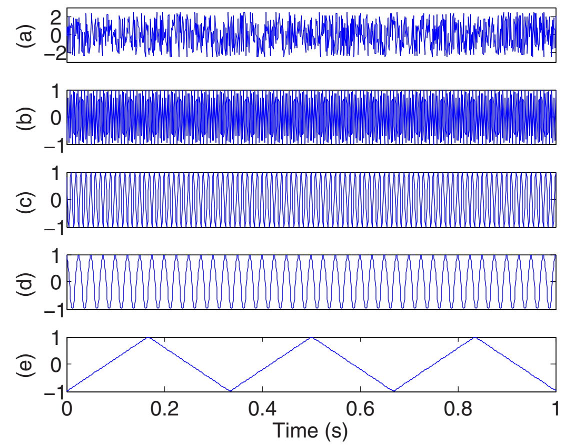

FIG. 5. (Color online) Original sources: (a) white Gaussian noise; (b) 150 Hz sin waveform; (c) 100 Hz sin waveform; (d) 40 Hz sin waveform; (e) 3 Hz triangular waveform.

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t),$$

$$x_2(t) = a_{21}s_1(t) + a_{22}s_2(t),$$

where s_1 is original LOD data and s_2 is white Gaussian noise. A normally distributed random number with a mean of 0 and a variance of 0.16 indicates the broad-band white Gaussian noise. These are randomly selected constants. Let us take the following mixing matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 2 & 9 \end{bmatrix}.$$

This interference of data transmission by noise is simulated in Fig. 6, in which the upper graph illustrates x_1 and lower figure, x_2.

Figure 7 indicates the decomposition results of the EMD from the LOD data by the noise disturbance. The ICA-EMD separates these two types of data into 11 components as show in Fig. 8. This result verifies Huang's study [28]; however, the research shows no noise condition. The first component is noise. Here c1 indicates the semimonthly tides. There are two meteorological tides with a cycle of 18.61 years. c2 represents the monthly tides, c3 shows the quasibimonthly tides, c5 is the semiannual cycle, c6 is the annual cycle, c7 is the quasibiennial cycle, and so forth. Notice the last component, c11, which is not an IMF but rather the trend. The comparison between Figs. 7 and 8 shows that EMD is unable to retrieve the period waveform in low-SNR condition due to the fact that EMD cubic spline is disturbed by noise and the correct components cannot be decomposed. We employ ICA as a preprocessor to remove the noise and then use EMD to decompose the components. Using the

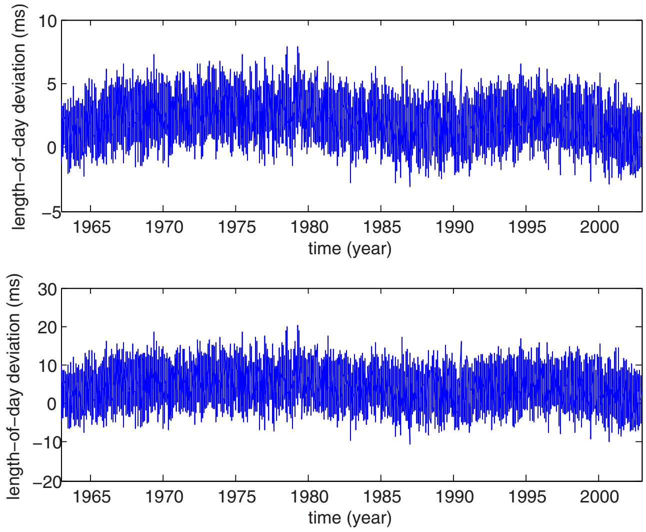

FIG. 6. (Color online) Simulation of this LOD data by the noise disturbance.

ICA-EMD, we can easily obtain the period waveform in low-SNR condition as shown in the results. The near periodic waveforms are crucial elements in the study of geophysics.

C. Several ICA constraints and conditions

For ICA to process data successfully, the following three conditions ought to first be satisfied. (1) The number of sensors should be greater than or equal to the number of sources. This will be improved considerably by ICA-EMD. (2) All the source signals must be statistically independent and have a non-Gaussian probability distribution. An exception is allowed for only one signal component. (3) Traditionally ICA can only analyze stationary data. (4) *A priori* knowledge of the probability distributions of the sources can be used in the cost function. In addition to these constraints, ICA also has other ambiguities: (1) the variances of the independent components cannot be determined. The signals separated by traditional ICA shows opposite phase and unequal amplitude. (2) The order will change because the order of the independent components cannot be determined.

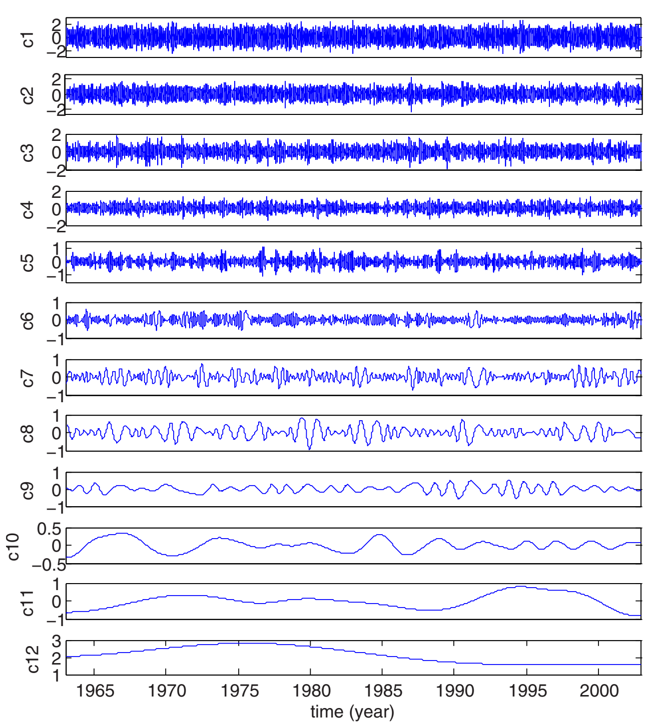

FIG. 7. (Color online) The resulting EMD component from the LOD data 1 by the noise disturbance.

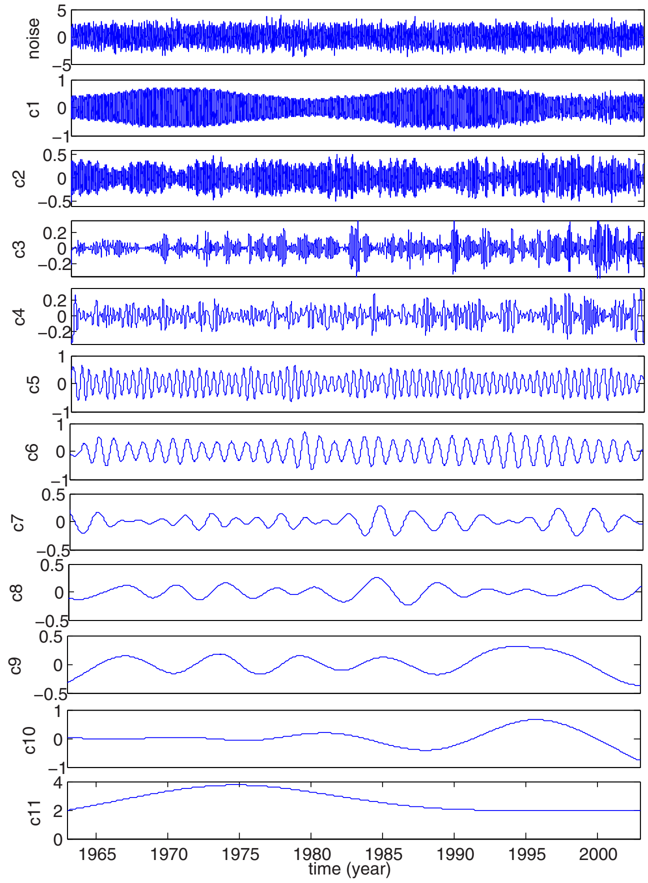

FIG. 8. (Color online) The LOD data separated by ICA-EMD.

V. CONCLUSION

ICA and EMD each have their own advantages and limitations. Thus, in this study, we propose a combination of ICA-EMD method that combines the advantages of both. Data with low SNR, which could not be dealt with in the past, may now be analyzed using the combined ICA-EMD method in several ICA constraints and conditions. This combined method is particularly useful for dealing with low-SNR conditions, a challenging area for data analysis. We know that noise is inevitable and ubiquitous. Some noise might be innocuous, others might be critical for the success or failure of a project. For example, noise interference in GPS could lead to plane crashes; a disturbance in radio transmission could obstruct global communication networks. The combined ICA-EMD analysis can offer a viable solution to noise removal and also lead to new discoveries.

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